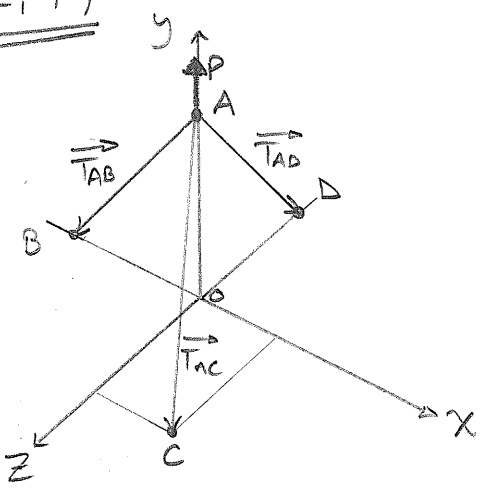
GNG 1505 SOLUTIONS DES EXERCICES SUGGÉRÉS

SEMAINE 3: (25 SEPT - 2 OCT) 2.99, 2.114, 2.125, 2.131

2.99



$T_{AB} = 259\,N$

Les forces en A sont: T_{AB}, T_{AC}, T_{AD} et P

$\vec{AB} = -4.2\vec{i} - 5.6\vec{j} \Rightarrow AB = 7.0\,m$

$\vec{AC} = 2.4\vec{i} - 5.6\vec{j} + 4.2\vec{k} \Rightarrow AC = 7.4\,m$

$\vec{AD} = -5.6\vec{j} - 3.3\vec{k} \Rightarrow AD = 6.5\,m$

$\vec{P} = P\vec{j}$

$\vec{T}_{AB} = T_{AB}\dfrac{\vec{AB}}{AB} = T_{AB}(-0.6\vec{i} - 0.8\vec{j})$

$\vec{T}_{AC} = T_{AC}\dfrac{\vec{AC}}{AC} = T_{AC}(0.32\vec{i} - 0.76\vec{j} + 0.16\vec{k})$

$\vec{T}_{AD} = T_{AD}\dfrac{\vec{AD}}{AD} = T_{AD}(-0.86\vec{j} - 0.51\vec{k})$

Condition d'équilibre: $\sum \vec{F} = 0$

$\Rightarrow \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{P} = 0$

$\Rightarrow (-0.6T_{AB} + 0.32T_{AC})\vec{i} + (-0.8T_{AB} - 0.76T_{AC} - 0.86T_{AD} + P)\vec{j}$

$+ (0.16T_{AC} - 0.51T_{AD})\vec{k} = 0$

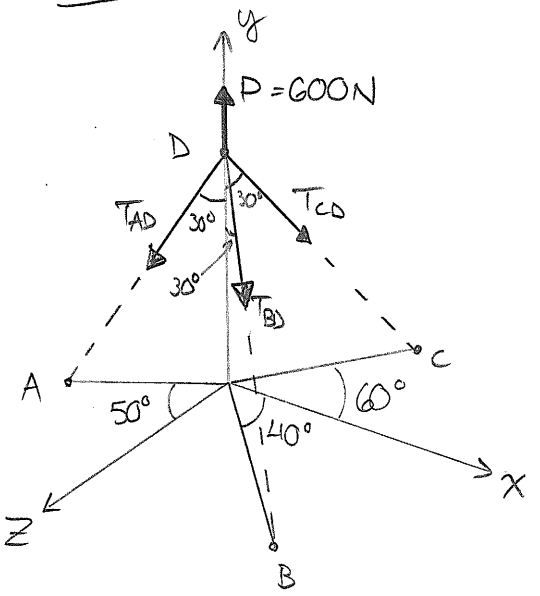
★ Les coefficients de $\vec{i}$, $\vec{j}$ et $\vec{k}$ doivent tous $= 0$

$\vec{i} \Rightarrow -0.6T_{AB} + 0.32T_{AC} = 0 \Rightarrow T_{AC} = 479.15\,N$

$\vec{k} \Rightarrow 0.16T_{AC} - 0.51T_{AD} = 0 \Rightarrow T_{AD} = 535.66\,N$

$\vec{j} \Rightarrow -0.8T_{AB} - 0.76T_{AC} - 0.86T_{AD} + P = 0 \Rightarrow \boxed{P = 1031\,N}$

2.114



On cherche : T_{AD}, T_{BD} et T_{CD}

$\Rightarrow$ Conditions d'équilibre :

$$\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$$

$$\Sigma F_x = 0 = -T_{AD}\sin 30 \sin 50 + T_{BD}\sin 30 \cos 40 + T_{CD}\sin 30 \cos 60 = 0$$

$\Rightarrow$ on divise par $\sin 30$ pour simplifier :

$$0 = -T_{AD}\sin 50 + T_{BD}\cos 40 + T_{CD}\cos 60 \quad \text{Eq 1}$$

$$\Sigma F_z = 0 = T_{AD}\sin 30 \cos 50 + T_{BD}\sin 30 \sin 40 - T_{CD}\sin 30 \sin 60$$

$\Rightarrow$ divise par $\sin 30$

$$0 = T_{AD}\cos 50 + T_{BD}\sin 40 - T_{CD}\sin 60 \quad \text{Eq 2}$$

De l'équation (1) :

$$T_{AD} = \frac{T_{BD}\cos 40}{\sin 50} + \frac{T_{CD}\cos 60}{\sin 50} = T_{BD} + 0{,}653\,T_{CD} \quad \text{Eq 3}$$

On substitut dans (2) $\left(T_{BD} + 0{,}653\,T_{CD}\right)\cos 50 + T_{BD}\sin 40 - T_{CD}\sin 60 = 0$

$$1{,}286\,T_{BD} = 0{,}446\,T_{CD} \quad \Rightarrow T_{CD} = 2{,}883\,T_{BD} \quad \text{Eq 4}$$

De retour dans (3) $T_{AD} = T_{BD} + 0{,}653(0{,}347\,T_{BD}) \Rightarrow T_{AD} = 2{,}883\,T_{BD}$ Eq 5

$$\Sigma F_y = 0 = P - T_{AD}\cos 30 - T_{BD}\cos 30 - T_{CD}\cos 30 \quad \text{Eq 6}$$

On substitut equations 4 et 5 dans 6

$$600 - 2{,}496\,T_{BD} - 0{,}866\,T_{BD} - 2{,}496\,T_{BD} = 0$$

$$600 = 5{,}858\,T_{BD}$$

$$\boxed{T_{BD} = 102{,}4\,N}$$

On retourne dans 4 et 5

$$\boxed{T_{AD} = 295{,}3\,N}$$

$$\boxed{T_{AC} = 295{,}3\,N}$$

2.125

$P = 341\,N\ \vec{j}$ — On cherche T_{AB} et Q

$y = 155\,mm$

$x = 200\,mm$

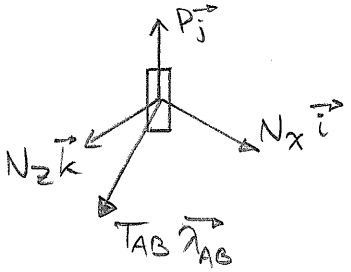
$AB = 525\,mm = \sqrt{x^2 + y^2 + z^2} = \sqrt{200^2 + 155^2 + z^2}$

$\Rightarrow z = 460\,mm$

$\overrightarrow{AB} = 200\vec{i} - 155\vec{j} + 460\vec{k}$

Vecteur unitaire $AB = \vec{\lambda}_{AB} = \frac{200}{525}\vec{i} - \frac{155}{525}\vec{j} + \frac{460}{525}\vec{k}$

Schéma corps isolé au manchon A :



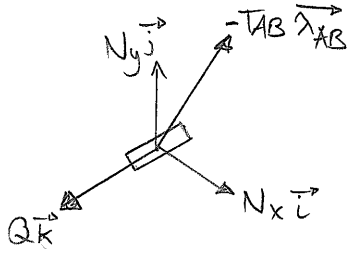
$\Sigma \vec{F} = 0 \quad \left(N_x + T_{AB}\frac{200}{500}\right)\vec{i} + \left(P - T_{AB}\frac{155}{525}\right)\vec{j}$

$+ \left(N_z + T_{AB}\frac{460}{525}\right)\vec{k}$

Coefficients de $\vec{i}$ $\vec{j}$ et $\vec{k}$ doivent $= 0$

en $\vec{j} \Rightarrow \quad P = T_{AB}\frac{155}{525}$ $\qquad \boxed{T_{AB} = 1155\,N}$

Schéma du corps isolé au manchon B :



$\Sigma \vec{F} = 0$

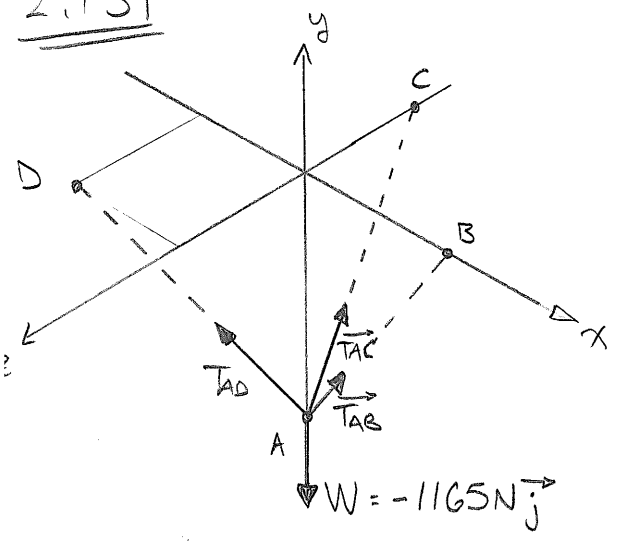
$0 = \left(N_x - T_{AB}\frac{200}{500}\right)\vec{i} + \left(N_y + T_{AB}\frac{155}{525}\right)\vec{j} + \left(Q - T_{AB}\frac{460}{525}\right)\vec{k}$

$\vec{k}$ coefficient $= 0$

$\therefore \quad Q - T_{AB}\frac{460}{525} = 0$

$\boxed{Q = 1012\,N}$

2.131



On cherche T_{AB}, T_{AC}, T_{AD}

$$\sum \vec{F} = 0 = \vec{T_{AB}} + \vec{T_{AC}} + \vec{T_{AD}} + \vec{W} = 0 \quad \text{Eq 1}$$

$$\vec{AB} = 450\vec{i} + 600\vec{j} \qquad AB = 750\,mm$$
$$\vec{AC} = 600\vec{j} - 320\vec{k} \qquad AC = 680\,mm$$
$$\vec{AD} = -500\vec{i} + 600\vec{j} + 360\vec{k} \qquad AD = 860$$

$$\vec{T_{AB}} = T_{AB}\vec{\lambda_{AB}} = T_{AB}\frac{\vec{AB}}{AB} = T_{AB}(0,6\vec{i} + 0,8\vec{j}) \quad \text{Eq 2}$$

$$\vec{T_{AC}} = T_{AC}\vec{\lambda_{AC}} = T_{AC}\frac{\vec{AC}}{AC} = T_{AC}(0,88\vec{j} - 0,47\vec{k}) \quad \text{Eq 3}$$

$$\vec{T_{AC}} = T_{AD}\vec{\lambda_{AD}} = T_{AD}\frac{\vec{AD}}{AD} = T_{AD}(-0,58\vec{i} + 0,70\vec{j} + 0,42\vec{k}) \quad \text{Eq 4}$$

- On substitut equations 2, 3 et 4 dans 1

$$(0,6\,T_{AB} - 0,58\,T_{AD})\vec{i} + (0,8\,T_{AB} + 0,88\,T_{AC} + 0,7\,T_{AD} - W)\vec{j} + (-0,47\,T_{AC} + 0,42\,T_{AD})\vec{k} = 0$$

- Les coefficients de $\vec{i}$ $\vec{j}$ et $\vec{k}$ sont tous égal à zero.

$$\vec{i} \Rightarrow 0,6\,T_{AB} - 0,58\,T_{AD} = 0 \Rightarrow T_{AB} = 0,969\,T_{AD} \quad \text{Eq 5}$$

$$\vec{k} \Rightarrow -0,47\,T_{AC} + 0,42\,T_{AD} = 0 \Rightarrow T_{AC} = 0,8895\,T_{AD} \quad \text{Eq 6}$$

- On substitut eqns 5 et 6 dans:

$$\vec{j} \Rightarrow 0,8\,T_{AB} + 0,88\,T_{AC} + 0,7\,T_{AD} - W = 0$$

$$0,8(0,969\,T_{AD}) + 0,88(0,8895\,T_{AD}) + 0,7\,T_{AD} = W$$

$$W = 2,2578\,T_{AD} \Rightarrow \boxed{T_{AD} = 516\,N}$$

- On retourne dans eqns 5 et 6 $\Rightarrow$ $\boxed{T_{AB} = 500\,N}$ $\boxed{T_{AC} = 459\,N}$